# CUSTOMER LIFETIME VALUE: MARKETING MODELS AND APPLICATIONS

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**ABSTRACT**

Customer lifetime value has been a mainstay concept in direct response marketing for many years, and has been increasingly considered in the field of general marketing. However, the vast majority of literature on the topic (a) has been dedicated to extolling its use as a decision-making criterion; (b) has presented isolated numerical examples of its calculation/determination; and (c) has considered it as part of the general discussions of profitability and discussed its role in customer acquisition decisions and customer acquisition/retention trade-offs. There has been a dearth of general modeling of the topic. This paper presents a series of mathematical models for determination of customer lifetime value. The choice of the models is based on a systematic theoretical taxonomy and on assumptions grounded in customer behavior. In addition, selected managerial applications of these general models of customer lifetime value are offered.

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## INTRODUCTION

Since the early eighties, the field of marketing has undergone a major directional change in both its theory and practice: a turn toward relationship marketing (Morgan & Hunt, 1994). At the core of relationship marketing is the development and maintenance of long-term relationships with customers, rather than simply a series of discrete transactions, achieved by creating superior customer value and satisfaction. Ideally, a loyalty that benefits both parties is fostered. One pitfall of this growing concern to maintain strong and long-lasting relationships, however, is to do it at the expense of profitability. Overly enthusiastic with the concept, many practitioners have gotten involved in losing relationships. Relationship marketing is costly. It might not pay to maintain long-term relationships, at least not all the time and not with all customers. Customers with low switching costs and short time-horizons might not be financially attractive to the firm (Jackson, 1985).

Ultimately, marketing is the art of attracting and keeping profitable customers (Kotler & Armstrong, 1996). A company should not try to pursue and satisfy every customer. What makes a customer profitable? Kotler and Armstrong (1996) define a profitable customer as ''a person, household, or company whose revenues over time exceed, by an acceptable amount, the company costs of attracting, selling, and servicing that customer.'' This excess is called customer lifetime value (CLV).

Customer lifetime value should be an important construct in designing and budgeting a number of marketing decisions such as customer acquisition programs (Dwyer, 1989). Recognizing its importance, many researchers in direct marketing have studied CLV and its managerial applications (Dwyer, 1989; Hughes & Wang, 1995; Keane & Wang, 1995; Wang & Splegel, 1994). A growing interest in CLV is expected in other marketing areas for two reasons. First, at a time when marketing methods are becoming more interactive, from frequent-user-club services to web pages, it is not surprising that marketing talk begins to sound like direct-marketing talk (Blattberg & Deighton, 1996). Second, changes in technology make it feasible to understand and track customer behaviors in ways that were impractical, or even impossible, in the past (Jackson, 1995).

Previous research in CLV has extolled the virtue of its use in a variety of marketing decision problems, primarily focusing on acquisition decisions or the acquisition/retention cost trade-off (Blattberg & Deighton, 1996; Wang & Splegel, 1994). Determining or calculating CLV was done solely by considering specific numerical cases. Researchers considered a particular setting, with specific input parameters, and computed CLV to use it in the decision-making problem prompting the CLV determination.

In this paper, a systematic general approach to the computation of CLV is offered. General mathematical models are provided to calculate CLV in a variety of typical cases. The major contribution of this paper is that it is less context-specific than previous discussions of CLV, and that it provides general mathematical formulations of CLV, while additionally tying together the specific assumptions underlying a formulation and, indeed, the formulation. Though not exhaustive, the cases treated deal with the large majority of typical practices. The choice of cases is based on both a systematic theoretical taxonomy and on assumptions grounded in customer behavior.

In addition to the introduction, this paper has four sections. First, we introduce a general way to determine CLV. Second, we treat five general cases, offering a mathematical model to compute CLV in each case. Each general case is followed by a numerical example. Third, we discuss some managerial applications of the use of a general model of CLV. As an illustration, we consider an example in which a general model of CLV is used to optimize the allocation of a promotional budget between Acquisition and Retention. Finally, we offer conclusions and suggest areas for future research.

## DETERMINATION OF CLV

Determining the CLV, or economic worth of a customer, is, in principle, a straightforward exercise. To calculate CLV, project the net cash flows that the firm expects to receive from the customer over time. Next, calculate the present value of that stream of cash flows. In practice, however, estimating the net cash flows to be received from that customer can be a very challenging task. The questions to be answered before making the necessary computations are not always easy to handle. Carpenter (1995) calls for taking a holistic view of what may come of the relationship with the customer while addressing this issue. Answers to questions such as, How many customers you can attract given specific acquisition spending? How large will the initial sale to a customer be? What is the probability that a customer will buy additional products or services from the company over time? How does this probability change with the amount spent on promotion? When will a customer stop buying from the company for good? are used as inputs in the computation of CLV.

## TYPES OF CUSTOMER BEHAVIOR

Jackson (1985) groups industrial buyers into two major categories: *lost-for-good,* and *always-a-share.* Her lost-for-good model assumes that a customer is either totally committed to the vendor or totally lost and committed to some other vendor. In the second model, always-a-share, the customer can easily experiment with new vendors. Switching costs constitute a major factor in implying one behavior or the other. Dwyer (1989) applied Jackson's taxonomy in direct marketing and showed its implications for lifetime valuation. A customer retention model is used to model lost-for-good situations. In this model, a retention rate (or retention probability) is estimated, traditionally based on historical data. The retention rate is ''the chance that the account will remain with the vendor for the next purchase, provided that the customer has bought from that vendor on each previous purchase'' (Jackson, 1985, p. 18).

A customer migration model characterizes the always-a-share case. In it, the recency of last purchase is used to predict the possibility of repeat purchase in a period. The argument that one may use purchase history, including recency, to predict repeat purchase behavior is plausible. DuWors and Haines (1990) use event history analysis to measure brand loyalty. In the customer retention model, a customer who stops dealing with a company is considered as lost-for-good. Returning customers are therefore treated as new ones. While this model might be more applicable in cases where switching costs are higher and customer commitment is a long-term one, other cases, where customers may discontinue their purchase of a particular product or brand only temporarily, also exist. A migration model is likely more applicable in such cases.

The CLV models in this paper include typical cases of customer behavior. The two models offered by Dwyer (1989) are considered. Cases 1, 2a and 2b, 3, and 4 below address customer retention situations. Case 5 deals with a customer migration model. Case 1 is the simplest and assumes yearly cycles of purchase. In many industries, the relevant purchase cycle is not one year (Reichheld, 1996). Cases 2a and 2b are a direct extension of case 1, in which the cycle is assumed to be shorter 2a or longer 2b than one year.

Profits per customer are not necessarily constant per cycle. A major advantage in retaining your customers is that the profits generated by them tend to accelerate over time. Reichheld and Sasser (1990) report examples of accelerating profits in credit card, industrial distribution and auto servicing. Reichheld (1996, p. 39) attributed the acceleration in customers' profits to four reasons. First, revenues from customers typically grow over time. For example, customers who newly acquire a credit card use it slowly at the beginning; in the second year, and subsequently, if they stay with the company/card, they become more accustomed to using the credit card and balances grow. Second, old (i.e., existing) customers are more efficient to serve and this usually results in costs savings. They do not request services the company does not have. Their familiarity with the company's products makes them less dependent on its employees for advice and help. Third, satisfied customers act as referrals who recommend the business to others (recommending, in addition, to themselves—i.e., cross selling or buying). Fourth, in some industries, old customers pay effectively higher prices than new ones. This is sometimes due to the trial discounts available only to brand-new customers. Though rare, one may also think of cases where profits per customer decrease per cycle. Cases 3 and 4 specifically address situations where profits per customer change over time. In case 3 cash flows are assumed to be discrete. In case 4, we drop this assumption in order to model CLV in cases with continuous cash flows.

## CLV MODELS

The focus in this paper is to determine the (accumulated and appropriately discounted) net *contribution margin* achieved per customer, once acquired. This focus has two implications. First, acquisition costs, the costs incurred to attract (i.e., acquire) a customer, while obviously an important input value for a variety of decision-making contexts, are not specifically considered in our determination of CLV. Managers can, however, consider the value computed in this paper as the maximum value they are willing to incur as acquisition costs. Acquisition costs exceeding this value indicate the existence of unprofitable customers. Second, fixed costs are not considered in this model. This treatment is in line with other research studies in direct marketing (Dwyer, 1989; Hughes and Wang, 1995; Wang and Spiegel, 1994). To compute CLV, we discount the difference between the revenues and both *cost of sales* and promotion expenses incurred to retain customers. Cost of sales includes both the cost of goods sold, and the cost of order processing, handling, and shipping (David Shepard Associates, Inc., 1995, p. 258). Promotion costs incurred to retain existing customers, such as sending personalized greeting cards and gifts, and general promotional expenditures, excluding those directly oriented toward acquisition, are referred to as retention costs. (Note that we view general image advertising and other routine promotional campaigns as enhancing retention. They may also, of course, enhance acquisition. By including all promotional costs except those very specifically oriented toward acquisition [e.g., a direct mail campaign in which existing customers have been purged], we achieve a CLV determination which is conservative in terms of being compared to acquisition costs.)

### *ase 1*

We start with a simple case to illustrate the concept. In this case we assume that (1) sales take place once a year, (2) both yearly spending to retain customers and the customer retention rate remain constant over time, and (3) revenues achieved per customer per year remain the same. We shall relax assumptions (1), (3), and the fixed retention rate assumption in (2), in subsequent cases.

In all cases with constant yearly net contribution margin per customer (i.e., cases 1, 2a, 2b, and 5), we assume a specific timing of cash flows. Both revenues from sales and the corresponding cost of sales take place at the time of sale; the first sales transaction occurs at the time of the determination of CLV, which may be thought of as the moment of acquisition. All promotional expenses, except in case 2b, are approximated (relative to uniform dispersion) to occur at the middle of the purchase cycle. This assumption results in slightly different discounting of these two sets of cash flows as the models in the constant net contribution margin cases show.

**Notations:**

- *GC* is the (expected) yearly gross contribution margin per customer. It is, therefore, equal to revenues minus cost of sales.
- is the (relevant) promotion costs per customer per year.
- $n$ is the length, in years, of the period over which cash flows are to be projected.
- $r$ is the yearly retention rate, i.e., the proportion of customers expected to continue buying the company's goods or services in the subsequent year.
- $d$ is the yearly discount rate (appropriate for marketing investments).

An illustration of cash flows in this case follows:

Now
I—*—I—*—I—*—I......I—*—I
Year 1 Year 2 Year 3 Year $n$

where the $I$ denotes cash flows (both inflows and outflows) pertaining to sales transactions, i.e., $GC$. On the other hand, an * indicates the approximate timing of promotional expenses. In the above case,

$$\text{CLV} = \{GC * \sum_{i=0}^{n} [r^i/(1 + d)^i]\} - \{ \quad * \sum_{i=1}^{n} [r^{i-1}/(1 + d)^{i-0.5}]\} \quad (1)$$

The length of the projection period, $n$, highly depends on the industry. Carpenter (1995) argues that looking beyond 5 years involves too much guesswork in high-technology industries. Dwyer (1989) considers $n$ to be 5 because the preponderance of lifetime value accrues in the first 4 or 5 years due to the shrinkage in the account base and the heavy discounting. One may, however, be interested in longer periods, especially in the case of durable products.

The $GC$ and cash flows are discounted differently because, as mentioned previously, they are assumed to take place at two different time instants. The 0.5 in equation (1) reflects the approximation of the promotion expenses to all occur at the middle of each purchase cycle.

**Numerical Example** A typical example of this case could be an insurance company trying to estimate its CLV. Suppose that the company pays, on average, $50 per customer yearly on promotional expenses. The yearly retention rate is 75%. The period of cash flows projection is 10 years. The yearly gross contribution per customer is expected to amount to $260. An appropriate discount rate for marketing activities is 20%. Then,

$$\begin{aligned}\text{CLV} &= \{260 * \sum_{i=0}^{10} [(.75)^i/(1 + .2)^i]\} \\ &\quad - \{50 * \sum_{i=1}^{10} [(.75)^{i-1}/(1 + .2)^{i-0.5}]\} \\ &= \$568.78\end{aligned}$$

***ase 2***

We relax the assumption of the first case that sales occur annually. The following cases are concerned with time periods that are longer or shorter than 1 year. The time periods are, however, still assumed to be equal in length.

**Case 2a** We consider first the case where sales occur more frequently than once a year. Let $p$ be the number of cycles (i.e., transactions or sales) per year. For instance, $p$ is equal to 2 for semiannual purchases/sales, and is equal to 4 in cases where sales occur quarterly; that is, $p$ = 12 divided by the cycle time in months. Then,

$$\text{CLV} = \{GC' * \sum_{i=0}^{pn} [(r')^i/(1 + d)^{i/p}]\} - \{ \quad ' * \sum_{i=1}^{pn} [(r')^{i-1}/(1 + d)^{(i-0.5)/p}]\} \quad (2)$$

where

$GC'$ is the (expected) gross contribution margin per customer per sales cycle,
$'$ is the promotion costs per customer per sales cycle, and
$r'$ is the retention rate per sales cycle.

As in case 1 above, $d$ is the yearly discount rate (appropriate for marketing investments). The number of periods is $n * p$; while it is not necessary that $p$ be an integer (e.g., $p$ = 2.4 for 5 purchase cycles per year), we assume that $n$ is such that $n * p$ is an integer; this simply corresponds with the projection period's not concluding in the middle of a purchase cycle. The power of $(1 + d)$ is divided by the number of periods per year because $d$ is indeed (still) the annual discount rate. The adoption of a nonannual discount rate would imply a change in the financial market; that is not the case here. The

0.5 is again used in the equation because promotion expenditures in a cycle are assumed to occur in the middle of that cycle.

**Numerical Example** A typical example of this case could be a health club trying to estimate its CLV. Suppose that customers subscribe for services on semiannual basis. The company pays $25 per customer semiannually on promotion. The semiannual retention rate is 80%. The period of cash flows projection is $n = 4$ years. The gross contribution margin per semiannual subscription amounts to $125. An appropriate discount rate for marketing activities is 20%. Then, based on equation (2),

$$\begin{aligned} \text{CLV} &= \{125 * \sum_{i=0}^{8} [(.8)^{i}/(1 + .2)^{i/2}]\} \\ &\quad - \{25 * \sum_{i=1}^{8} [(.8)^{i-1}/(1 + .2)^{(i-0.5)/2}]\} \\ &= \$354.69 \end{aligned}$$

**Case 2b** In this case, sales/transactions occur less frequently than once a year. In cases of durables, replacements often occur only every few years. Let $q$ be the length of a cycle or the number of years between two consecutive sales. For example, if a car is leased every 3 years, then $q = 3$. Then,

$$\begin{aligned} \text{CLV} = &\{GC' * \sum_{i=0}^{n/q} [(r')^{i}/(1 + d)^{iq}]\} \\ &- \{\quad ' * \sum_{i=1}^{n} [(r')^{(i-1)/q}/(1 + d)^{i-0.5}]\} \end{aligned} \tag{3}$$

We assume in this case that promotion costs are approximated to occur at the middle of each year of the cycle, and again, sales and the corresponding cost of sales occur once per purchase cycle, with the first transaction taking place at the time of the acquisition/determination of CLV. Cash flows are illustrated as follows (note that the number of purchase cycles equals $n/q$):

Now
I—*—x—*—x . . .
Year 1   Year 2
x—*—I—*—x—*—x . . . x—*—I . . .
Year $q$   Year $q+1$
I—*—x—*—x . . . x—*—I
Year $n$

where the I, the beginning of purchase cycles, denotes cash flows (both inflows and outflows) pertaining to sales transactions, i.e., $GC$. On the other hand, the * shows the approximate timing of promotional expenses (assumed to be the middle of each year). One may relax the assumptions concerning the timing of cash flows without major changes in the model. The value of $r'$ pertains to a full cycle.

**Numerical Example** Consider the case of a car dealership where customers lease cars for 3 years. The company pays $95 per customer on promotion, yearly. The cyclical retention rate is (only) 30%. The average gross contribution margin per car lease per cycle is $7,000. An appropriate discount rate is 20%. The company wants to project its CLV for the next 12 years (12/3 = 4 purchase cycles). In this case,

$$\begin{aligned} \text{CLV} &= \{7{,}000 * \sum_{i=0}^{4} [(.3)^{i}/(1 + .2)^{3i}]\} \\ &\quad - \{95 * \sum_{i=1}^{12} [(.3)^{(i-1)/3}/(1 + .2)^{i-0.5}]\} \\ &= \$8{,}273.31 \end{aligned}$$

(Note that the leasee perhaps pays the lease cost monthly; however, the lessor receives payment up front, irrespective of the leasee's financing choice.) To use equation (3), $n/q$, as noted earlier for $p * n$, should be an integer value. Again this corresponds with the fact that the projection period does not conclude in the middle of a purchase cycle. The values of $p$ and $q$ are not chosen by the manager, but are based on the nature of the product and its related purchase cycle. However, the manager can always set $n$, the number of years over which he

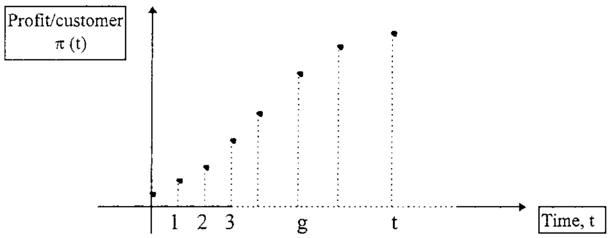



**FIGURE 1**
Accelerating profit per customer—Discrete cash flows

wants to project cash flows, in a manner that leads to the required integer value. Given that the manager is likely interested in estimating CLV for a certain number of complete purchase cycles, this restriction is generally a minimal one.

### *ase 3*

In both cases 1 and 2 (a and b) we assume that the gross contribution margin (*GC*) per purchase cycle and relevant promotion costs (  ) per year per customer remain constant. In cases 3 and 4, we address situations with *GC* and   per customer potentially nonconstant over time. For example, in case of accelerating profits, one could have an S-shaped customer lifecycle profit (*GC* −  ) pattern similar to that implied in Figure 1 (and later plotted, as a continuous curve in Figure 2, case 4).

To estimate customer lifetime value, one needs, in this situation, to estimate the customer's profit function over time, $\pi(t)$. The curve in Figure 1 is a function that first grows at an increasing rate (up to point $g$), then at a decreasing rate, and has an upper asymptote. The upper asymptote reflects a ceiling that profits are not expected to exceed. (Note: $\pi(t)$ represents profit *at* time $t$, and is not cumulative). The following function might serve as an example of an approximation to the profit curve of Figure 1:

$$\left[\begin{array}{ll} \pi(t) = \pi_1(t) = ht^2 + v & \text{for } t \le g \\ \pi(t) = \pi_2(t) = \pi_1(g) & \\ \qquad + [N(1 - e^{-t+g})] & \text{for } t > g \end{array}\right. \tag{4}$$

where $h$, $g$, $v$, and $N$ are all positive constants. The rate at which profit grows over time affects the value of $h$. The value $g$ is the time at which the inflection point in the profit curve occurs, and $(\pi_1(g) + N)$ is the expected ceiling for profits reached asymptotically. Companies typically use historical data to estimate those values. The intercept $v$ is the company's gross contribution margin from the first sale. Sometimes, this value is not especially high; in some cases, it might even be near zero, and on rare occasion, negative (recall—we are not including acquisition costs; if we did include them, first year "profit" would often be negative).

The CLV in this case is computed as follows:

$$\text{CLV} = \sum_{t=0}^{n} \pi(t) * [r^t/(1 + d)^t] \tag{5}$$

where $\pi(t)$ is the profit per customer in year $t$. In this case, as in case 1, we assume a yearly cycle. Applications to shorter or longer periods, as in cases 2a and 2b, are straightforward.

Based on equations (4) and (5), with $g$ being an integer, we have

$$\begin{aligned} \text{CLV} = \sum_{t=0}^{g} \{[ht^2 + v] * [r^t/(1 + d)^t]\} \\ + \sum_{t=g+1}^{n} \{[[hg^2 + v] + [N(1 - e^{-t+g})]] \\ * [r^t/(1 + d)^t]\} \end{aligned} \tag{6}$$

Note that the case of accelerated profits was purposely chosen as the example of cases where profits per customer change over time. This choice is based on the fact that the case of increased profit over time was singled out by previous researchers as a frequently occuring one (Reichheld, 1996; Reichheld & Sasser, 1990). The same general approach of equation (5) can be applied to other cases where change in profit over time exhibits other patterns, including decreasing ones.

As noted, in the above model, *profits* means *net contribution margin.* For simplicity, we did not separate gross contribution margin, *GC,* from promotional expenses,  . Given separate functions for *GC* and  , one can follow the same procedure applied in this case to discount *GC* and separately, and then combine them, as in the previous cases.

**Numerical Example** Consider the case of a credit card company that expects its profit per newly acquired customer to accelerate over time. Profit per customer starts at a low level of \$20. This profit is, however, expected to grow at an increasing rate until year 5. Afterward, profit will continue to grow, but at a decreasing rate. Profit is not expected to exceed a ceiling of \$200. The retention rate is 90%, and the discount rate is 20%. Profit per customer can be approximated as a function of time as follows:

$$\begin{bmatrix} \pi(t) = 4t^2 + 20 & \text{for } t \leq 5 \\ \pi(t) = 120 + [80(1 - e^{-t+5})] & \text{for } t > 5 \end{bmatrix}$$

Note that $\pi(5) = \$120$. If this company is projecting its cash flows for the next 8 years, then its CLV is equal to:

$$\sum_{t=0}^{5} \{(4t^2 + 20) * [(.9)^t/(1 + .2)^t]\}$$
$$+ \sum_{t=6}^{8} [(4 * 5^2 + 20) + [80(1 - e^{-t+5})]]$$
$$* [(.9)^t/(1 + .2)^t]$$
$$= \$212.163.$$

Cases with accelerating customer profit show the importance of retaining customers. Any change in retention rate is expected to have a greater effect on CLV when profit per customer is accelerating than cases when profit per customer is constant over time. This is mainly due to the fact that the compounded retention rate is multiplied by a growing profit value in computing CLV. Hughes and Wang (1995) show the dramatic effect of a lower retention rate on CLV in the case of credit card customers. In the example above, dropping the yearly retention rate from 90% to 80% results in a drop of 37% in CLV (new CLV = \$134.68).

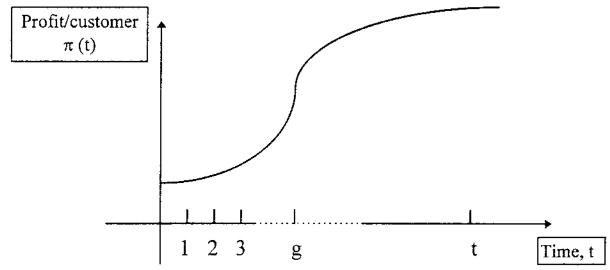



**FIGURE 2**
Accelerating profit per customer—Continuous cash flows

### *ase 4*

In all the cases considered so far, we have assumed discrete cash flows. In case 4 we address the situation of continuous cash flows. Cases of continuous cash flows, at least as a very close approximation, are common in practice. Daily consumed products, such as coffee or cigarettes, are likely to be well approximated by a continuous purchase pattern.

In addressing the case of continuous cash flows, we shall refer to the same profit function used in case 3 with profits per customer accelerating over time (i.e., nonconstant). Applications to other cases with constant (or uniform) cash flows are straightforward. The curve in Figure 2 illustrates the customer lifecycle profit (*GC* − ) pattern of case 4.

To model CLV in cases with continuous cash flows, one must generally use calculus. The summation function is replaced by an integral to reflect the substitution of an essentially discrete function by a continuous one. The limits of the integral reflect the time period over which the cash flows are projected, and, hence, the CLV is determined. To make this conversion (summation → integral) complete, one needs, however, to add to the integral the initial net cash (in)flow (*GC* at time zero, the time of the first sale). The separate addition of the first cash inflow is to allow for the fact that, in moving from a discrete function to a continuous function, the definite integral accumulates cash flows for $t$ periods, while the summation (discrete) representation is accumulating the initial cash inflow plus the cash flows of $t$ periods, a total of $(t + 1)$ inflows.

In general, in addition to the initial net cash

inflow (as explained above), CLV is the sum of the discounted profits per each period $t$, $\pi(t)$, taking retention rate into consideration. Therefore, CLV is the definite integral of the "$\pi(t) * r^t$" function discounted continuously, plus the initial $\pi(t)$ (i.e., $\pi[0]$). We should also note that, in this case, we still have a yearly discount rate, $d$. We therefore first need to compute a nominal annual rate, $d'$, that, when compounded continuously, is equivalent to the desired effective rate, $d$. For fractional periods, we have:
Effective Annual Rate

$$= (1 + \text{Nominal Annual Rate}/c)^c - 1 \qquad (7)$$

where $c$ is the number of compounding periods per year (Weston & Brigham, 1993, p. 232).

With continuous compounding, $c$ tends to $\infty$, and $(1 + \text{Nominal Annual Rate}/c)^c$ is, in the limit, equal to $e^{\text{Nominal Annual Rate}}$. Hence, equation (7) can also be expressed as: $d = e^{d'} - 1$. Therefore, $d' = \ln(1 + d)$. We also know that, to continuously discount a cash flow expected in period $t$ at a nominal rate of $d'$, we need to multiply this value by $e^{-td'}$ (Weston & Brigham, 1993, p. 233).

Based on the above discussion, we have:

$$\text{CLV} = \pi(0) + \int_0^n \pi(t) * r^t * e^{-td'} d(t),$$

with $d' = \ln(1 + d)$ or:

$$\text{CLV} = \pi(0) + \int_0^n \pi(t) * r^t * e^{-t\ln(1+d)} d(t)$$

$$= \pi(0) + \int_0^n \pi(t) * [r/(1 + d)]^t \, d(t) \qquad (8)$$

Based on equations (4) and (8), we have:

$$\text{CLV} = v + \int_0^g (ht^2 + v) * [r/(1 + d)]^t \, d(t)$$
$$+ \int_g^n [(hg^2 + v) + [N(1 - e^{-t+g})]]$$
$$* [r/(1 + d)]^t \, d(t) \qquad (9)$$

**Numerical Example** We refer to the same numerical example of case 3, with one major difference: Cash flows are continuous. Then,

$$\text{CLV} = 20 + \int_0^5 (4t^2 + 20) * [.9/(1 + .2)]^t \, d(t)$$
$$+ \int_5^8 [(4 * 5^2 + 20)$$
$$+ [80(1 - e^{-t+5})]] * [.9/(1 + .2)]^t \, d(t)$$
$$= \$213.13$$ (Recall: CLV in case 3 is equal to \$212.163)

### *ase 5*

The above cases assume a shrinking customer base over time, in which lost customers are treated as new ones if they return. In case 5, we use purchase history, particularly recency, to predict repeat purchase behavior. In his customer migration model, Dwyer (1989) uses the recency of last purchase to predict the probability of repeat purchase for the next period. For ease of presentation, we shall first use Dwyer's (1989) example to present the case under discussion. We shall then provide the necessary equations to compute CLV. The sales cycle is assumed to be annual. The length of the cycle is not, however, a critical factor in constructing the model. We shall drop from Dwyer's model data on costs and earnings, and focus on the most critical factor in the model: the number of customers per year.

The model uses empirical evidence of purchase recency to predict repeat purchase behavior. From past data, the purchase propensities of each recency cell have been estimated. Table 1 summarizes the probability of purchase for members of each recency cell.

Table 2 shows the number of customers over a 4-year period after acquisition, starting with a base of 1,000 customers.

Equation (7) shows how to compute $C_i$, the number of customers in year $i$:

$$C_i = \sum_{j=1}^{i} [C_{i-j} * P_{t-j} * \prod_{k=1}^{j} (1 - P_{t-j+k})],$$
$$\text{with } P_t = 0 \qquad (10)$$

For instance, the number of customers in year 4 in the example of table 2 above is:

**TABLE 1**
Purchase Probabilities—Customer Migration Model

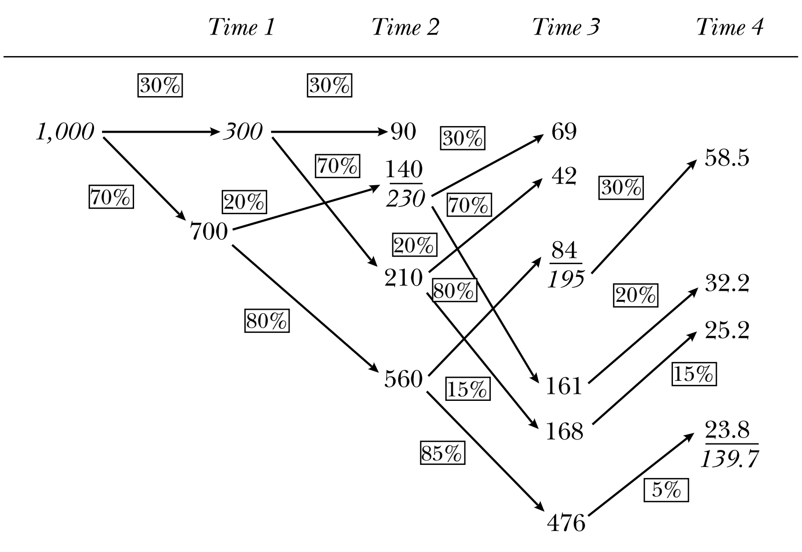
| *Recency Cell* | *Probability of Purchase ($P_{t-j}$) (for the current year, t)* |
|---|---|
| 1—If last purchase was in year ($t-1$) | .30 |
| 2—If last purchase was in year ($t-2$) | .20 |
| 3—If last purchase was in year ($t-3$) | .15 |
| 4—If last purchase was in year ($t-4$) | .05 |
| 5—If last purchase was in year ($t-5$) | .00 |

$$
\begin{aligned}
C_4 = {} & [C_{4-1} * P_{t-1} * (1 - P_{t-1+1})] \\
& + [C_{4-2} * P_{t-2} * (1 - P_{t-2+1}) * (1 - P_{t-2+2})] \\
& + [C_{4-3} * P_{t-3} * (1 - P_{t-3+1}) * (1 - P_{t-3+2}) \\
& * (1 - P_{t-3+3})] \\
& + [C_{4-4} * P_{t-4} * (1 - P_{t-4+1}) \\
& * (1 - P_{t-4+2}) * (1 - P_{t-4+3}) \\
& * (1 - P_{t-4+4})] \\
= {} & [C_3 * P_{t-1} * (1 - P_t)] \\
& + [C_2 * P_{t-2} * (1 - P_{t-1}) * (1 - P_t)] \\
& + [C_1 * P_{t-3} * (1 - P_{t-2}) * (1 - P_{t-1}) \\
& * (1 - P_t)] + [C_0 * P_{t-4} * (1 - P_{t-3}) \\
& * (1 - P_{t-2}) * (1 - P_{t-1}) * (1 - P_t)] \\
= {} & [195 * .3 * (1 - 0)] \\
& + [230 * .2 * (1 - .3) * (1 - 0)] \\
& + [300 * .15 * (1 - .2) * (1 - .3) * (1 - 0)] \\
& + [1{,}000 * .05 * (1 - .15) * (1 - .2) \\
& * (1 - .3) * (1 - 0)] \\
= {} & 58.5 + 32.2 + 25.2 + 23.8 \\
= {} & 139.7
\end{aligned}
$$

Then, in general, in the always-a-sale case, and applying the same assumptions of cash flow timing of case 1, the company CLV is computed as follows:

$$
\mathrm{CLV} = \{(GC) * \{C_0 + [\sum_{i=1}^{n} [\sum_{j=1}^{i} C_{i-j} * P_{t-j} * \prod_{k=1}^{j} (1 - P_{t-j+k})]] / (1 + d)^i\}
$$

**TABLE 2**
Number of Customers—Customer Migration Model



$$- \{ \quad * [[C_0/(1+d)^{0.5}] + [\sum_{i=1}^{n} \sum_{j=1}^{i} C_{i-j} * P_{t-j} * \prod_{k=1}^{j} (1 - P_{t-j+k})]]/(1+d)^{i+0.5}\}\}/C_0,$$

$$\text{with } P_t = 0, \quad (11)$$

where $C_0$ is the initial customer base at the time of the determination of CLV (acquisition). Note that, in this case, we assume that sales, and the corresponding cost of sales, take place once a year; the first transaction is at acquisition. Promotion expenses occur at the middle of each year.

## MANAGERIAL APPLICATIONS

Relationship marketing is the process of creating, maintaining, and enhancing strong, value-laden relationships with customers and other stakeholders (Kotler & Armstrong, 1996). Companies are realizing that when operating in mature markets and facing stiffer competition, the development of this profitable relationship with customers is a critical success factor. To know whether or not a relationship is profitable, one needs to be able to quantify this relationship. The general models presented in this paper serve as tools to quantify customer lifetime value, a construct that had been considered primarily in abstract terms (Carpenter, 1995).

Determining customer value can help managers in making decisions through determining the impact of different courses of action on the value of CLV. The use of general models makes the study of the impact a more systematic exercise. These models can, for example, be used to decide how much to spend on promotional campaigns. They are also able to be used to check the difference in profitability among various market segments.

The relationship between acquisition costs and rates, and retention costs and rates is worthy of consideration (Blattberg & Deighton, 1996). The CLV determination can help us determine the effect of adopting a marketing strategy, with its resulting acquisition and retention rates, costs, and trade-off. Also, one may use the models to determine the effects of some situational factors on customer equity. For example, the adoption of a price skimming strategy typically results in a lower acquisition rate. Though less in number, however, these acquired customers might show a higher degree of persistence in their choice of a seller. The comparison of expected CLV in the case of price penetration to that of price skimming enables managers to make a more informed pricing policy choice.

CLV determination can also be used to decide how to allocate promotional budgets between acquisition and retention spending. In the following paragraphs, we build on a study by Blattberg and Deighton (1996) and use a general model of CLV to formulate a nonlinear programming problem to optimize this allocation.

In a recent article, Blattberg and Deighton (1996) presented a procedure to determine the optimal acquisition and retention costs, based on the maximization of customer lifetime value. In their model,

$$\text{Customer Equity} = am - A + a * (m - R/r) * [r''/(1 - r'')] \quad \text{with } r'' = r/(1 + d), \quad (12)$$

where,

- $a$ is the acquisition rate (proportion of solicited prospects acquired), given a specific level of acquisition costs ($A$),
- $m$ is the margin (in monetary units) on a transaction,
- $A$ is the acquisition cost per customer,
- $R$ is the retention cost per customer per year,
- $r$ is the yearly retention rate, and
- $d$ is the yearly discount rate (appropriate for marketing investments).

The difference between customer equity and CLV is that customer equity takes acquisition cost into consideration. As mentioned earlier, we deliberately dropped acquisition cost from our CLV models, in order to clearly distinguish them from prospect models.

In this model, $a$ and $r$ are exponential functions of $A$ and $R$, respectively (described below). These functions are determined based on information provided by managers on (1) their current spending on acquisition (retention) and the resulting acquisition (retention) rate, and (2) the ceiling of their acquisition (retention) rate that would be achieved were there no limit on the amount spent on acquisition (retention). In their study, the acquisition rate $a$ = ceiling rate $* [1 - \exp(K_1 * A)]$ with the ceiling rate and $K_1$ being constant and determined from the managers' answers. The same applies to the retention rate where $r$ = ceiling rate $* [1 - \exp(K_2 * R)]$. To determine optimal acquisition costs, customer value in the first year ($= a * m - A$) is maximized. To decide on retention costs, the acquisition rate, $a$, resulting from the maximization of the first-year customer value is first plugged in equation (12) above. Then, the optimal spending on retention, $R$, is the value resulting in the maximization of customer equity as per equation (12).

Different acquisition methods may result in different retention rates in the future. Thus, arguing that measuring the success of customer acquisition efforts solely in terms of response (or acquisition) rates might not be appropriate, Wang and Splegel (1994) propose a model to capture the dynamics/interaction of acquisition and retention rates. Methods having dissimilar acquisition rates might also have dissimilar retention rates, most often in the opposite direction. In fact, the functions relating acquisition spending to acquisition rates, and those relating retention spending to retention rates might be illustrated in Figure 3. The first method, subscripted ''1'', could be offering a free premium with the first order; this could result in a higher acquisition rate, $a_1$, but a lower retention rate, $r_1$, relative to a second method, subscripted as ''2'', e.g., advertising. The manager is faced with a trade-off between the two rates, as determined by choice of method (or allocation between each method).

Referring to equation (12), one may extend the procedure in Blattberg and Deighton (1996), by the use of mathematical modeling to decide on which promotional method to choose. The aim would be to maximize the customer equity. Indeed, to determine the customer equity that would result from each of methods 1 and 2, replace $a$ and $r$ in equation (12), first, by $a_1$ and $r_1$, and, then, $a_2$ and $r_2$, respectively. The manager will choose the method with the higher customer equity.

Blattberg and Deighton (1996) used their models to determine optimal acquisition costs ($A$) and optimal retention costs ($R$). They, however, did not consider a limited availability of funds, and did not address the allocation/tradeoff between acquisition and retention spending. In the real world, managers often operate with limited budgets. The promotional budget set for a particular year should, in general, be allocated between acquisition and retention spending. One can extend their general model of customer equity to optimize the allocation of the promotional budget. Indeed, the problem to be solved is a nonlinear programming problem, where the objective function to be maximized is:

$$\begin{aligned}\text{Customer Equity} = (am - A) + a * (m - R/r) \\ * [r''/(1 - r'')], \\ \text{with } r'' = r/(1 + d),\end{aligned}$$

Subject to the following constraints:

**1** $A + R \leq$ Total promotional budget
**2** $A \geq 0$
**3** $R \geq 0$

Recall that $a$ and $r$ are *nonlinear* functions of $A$ and $R$, respectively. The objective function has 2 decision variables: $A$ and $R$.

## CONCLUSION AND SUGGESTIONS FOR FUTURE RESEARCH

The basic insight that comes from looking at the economics of customer lifetime value is that one begins to view customers in terms of ongoing relationships, rather than transactions. Optimization techniques focusing on a short-term impact of marketing strategies do not necessarily draw a correct picture for managerial use.

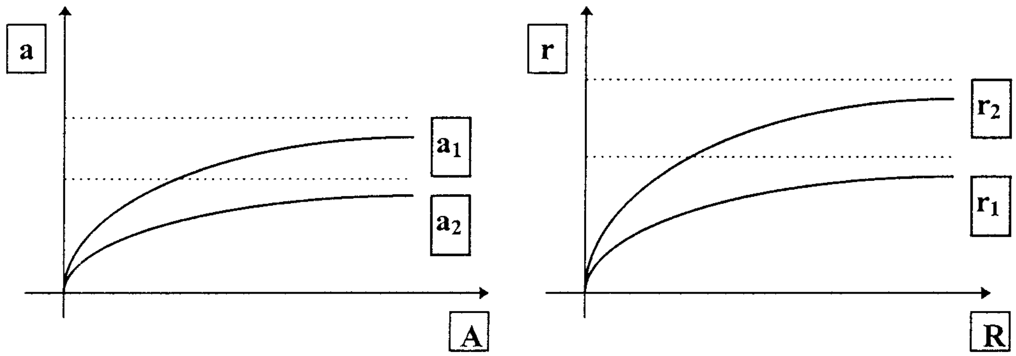



**FIGURE 3**
Trade-off between acquisition rates and retention rates

This paper adds to a stream of research that calls for a richer managerial analysis of a variety of key marketing decision problems, through offering tools that make it practical for managers to develop long-term perspectives.

Though typical, the cases addressed in this paper are far from being exhaustive. The study of more complex cases, some of which being combinations of the cases investigated in this paper, is needed. Modeling customer lifetime value of other real world situations can be managerially helpful. For example, one might be interested in studying this construct in hybrid cases of repeat purchase where, for instance, a customer might keep an account with a company while significantly dropping the value of this account. Reichheld (1996) refers to the viewing of these cases as a subset of simple repeat purchase models as a misguided effort with respect to measurement of customer value, and a pitfall to be avoided. One may also desire to reflect the effect of inflation, on both cash inflows and outflows, in the model.

The study of customer lifetime value naturally ties in with studies on brand loyalty. It can be argued that one cannot address the first construct in a fully useful manner without relating it to the second one. Researchers have generally drawn a clear distinction between the prospect of repeat purchase and the construct of brand loyalty (Jacoby & Chestnut, 1978; Jacoby & Kyner, 1973; Jones & Sasser, 1995). They have also used measures of brand loyalty that encompass attitudes toward products (Day, 1969; DuWors & Haines, 1990). Despite these facts, models used to measure CLV account for the repeat purchase probability without any consideration of attitudinal factors. Though the repeat purchase probability is ultimately the value that one needs to use to determine CLV, the development of mathematical models to measure CLV that take into consideration factors underlying the repeat purchase behavior (and, thus, customer satisfaction and existence of competition, for examples) remains a challenge for future researchers.

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